Y13FM Pure Maths

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| **Series**  Know the difference between a sequence and a series.  Know the meaning of the word *converge* when applied to either a sequence or a series.  Be able to sum series using partial fractions.  Be able to find the Maclaurin series of a function, including the general term.  Know that a Maclaurin series may converge only for a restricted set of values of *.* x  Be able to recognise and use the Maclaurin series of standard functions: , , , and |
| **Polar Coordinates**  Understand and use polar coordinates and be able to convert from polar to cartesian coordinates and vice-versa.  Be able to sketch curves with polar equations where *r* is given as a function of *θ*.  Be able to find the area enclosed by a polar curve. |
| **Matrices**  Be able to find the determinant and inverse of a 3×3 matrix without a calculator. |
| **Complex Numbers**  Understand and use de Moivre's theorem.  Be able to apply de Moivre's theorem to finding multiple angle formulae and to summing suitable series.  Understand the definition and hence the form .  Know that every non-zero complex number has *n* distinct *n*th roots, and that on an Argand diagram these are the vertices of a regular *n*-gon.  Know that the distinct *n*th roots of are:  , for  Be able to explain why the sum of all the *n*th roots is zero.  Understand the effect of multiplication by a complex number on an Argand diagram.  Be able to represent complex roots of unity on an Argand diagram.  Be able to apply complex numbers to geometrical problems. |
| **Vectors**  Be able to use the vector product in component form  Be able to use the alternative form for the vector product. Know the significance of  Be able to form and use the equation of a line in 3- D.  Be able to calculate the angle between two lines.  Know the different ways in which two lines can intersect or not in 3-D space.  Be able to determine whether two lines in three dimensions are parallel, skew or intersect, and to find the point of intersection if there is one.  Be able to find the distance between two parallel lines and the shortest distance between two skew lines.  Be able to find the intersection of a line and a plane.  Be able to calculate the angle between a line and a plane.  Be able to find the distance from a point to a line in 2 or 3 dimensions.  Be able to find the distance from a point to a plane. |
| **Calculus**  Evaluate improper integrals where either the integrand is undefined at a value in the interval of integration or the interval of integration extends to infinity.  Be able to derive formulae for, and calculate the volumes of, the solids generated by rotating a plane region about the *x*-axis or the *y*-axis.  Understand and evaluate the mean value of a function.  Be able to use the method of partial fractions in integration, including where the denominator has a quadratic factor of form and one linear term.  Understand the definitions of inverse trigonometric functions.  Be able to differentiate inverse trigonometric functions.  Recognise integrals of functions of the form and and be able to integrate related functions by using trigonometric substitutions. |
| **Hyperbolic Functions**  Understand the definitions of hyperbolic functions, know their domains and ranges and be able to sketch their graphs.  Understand and use the identity =1  Be able to differentiate and integrate hyperbolic functions.  Understand and be able to use the definitions of the inverse hyperbolic functions and know their domains and ranges.  Be able to derive and use the logarithmic forms of the inverse hyperbolic functions.  Recognise integrals of functions of the form and and be able to integrate related functions by using substitutions. |
| **Differential Equations**  Understand how to introduce and define variables to describe a given situation in mathematical terms.  Be able to relate 1st and 2nd order derivatives to verbal descriptions and so formulate differential equations.  Know the language of kinematics, and the relationships between the various variables.  Know Newton's 2nd law of motion. (F=ma)  Use differential equations in modelling in kinematics and in other contexts.  Know the difference between a general solution and a particular solution. Be able to find both general and particular solutions.  Recognise differential equations where the integrating factor method is appropriate.  Be able to find an integrating factor and understand its significance in the solution of an equation.  Be able to solve an equation using an integrating factor and find both general and particular solutions.  Be able to solve differential equations of the form , using the auxiliary equation.  Understand and use the relationship between different cases of the solution and the nature of the roots of the auxiliary equation.  Be able to solve differential equations of the form , by solving the homogeneous case and adding a particular integral to the complimentary function.  Be able to find particular integrals. Understand the relationship between different cases of the solution and the nature of the roots of the auxiliary equation.  Be able to solve the equation for simple harmonic motion, , and be able to relate the solution to the motion.  Be able to model damped oscillations using 2nd order differential equations.  Be able to interpret the solutions of equations modelling damped oscillations in words and graphically.  Analyse and interpret model situations with one independent variable and two dependent variables which lead to coupled 1st order simultaneous linear differential equations and find the solution. |

Y13FM Mechanics

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| **Dimensional Analysis**  Be able to find the dimensions of a quantity in terms of M, L, T.  Understand that some quantities are dimensionless.  Be able to determine the units of a quantity by reference to its dimensions.  Be able to change the units in which a quantity is given.  Be able to use dimensional analysis to check the consistency of a relationship.  Use dimensional analysis to determine unknown indices in a proposed formula.  Use a model based on dimensional analysis. |
| **Forces: Friction and Moments**  Understand the language relating to forces.  Understand that the value of the normal reaction depends on the other forces acting and why it cannot be negative.  Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to represent the situation in an appropriate force diagram.  Understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal contact force (normal reaction).  Understand that the frictional force may be modelled by and that friction acts in the direction to oppose sliding. Model friction using when sliding occurs.  Be able to derive and use the result that a body on a rough slope inclined at an angle to the horizontal is on the point of slipping if  Be able to apply Newton's Laws to situations involving friction.  Be able to resolve a force into components and be able to select suitable directions for resolution.  Be able to find the resultant of several concurrent forces by vector addition.  Know that a particle is in equilibrium under a set of concurrent forces if and only if their resultant is zero.  Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.  Be able to formulate and solve equations for equilibrium by resolving forces in suitable directions, or by drawing and using a polygon of forces.  Be able to draw a force diagram for a rigid body.  Understand that a system of forces can have a turning effect on a rigid body.  Know the meaning of the term couple.  Be able to calculate the moments about a fixed axis of forces acting on a body.  Be able to calculate the moment of a couple.  Understand and be able to apply the conditions for equilibrium of a rigid body.  Be able to identify whether equilibrium will be broken by sliding or toppling. |
| **Energy and Power**  Understand the language relating to work, energy and power.  Be able to calculate the work done by a force which moves along its line of action.  Be able to calculate the work done by a force which moves at an angle to its line of action.  Be able to calculate kinetic energy.  Be able to calculate gravitational potential energy.  Understand when the principle of conservation of energy may be applied and be able to use it appropriately.  Understand and use the work-energy principle.  Understand and use the concept of the power of a force as the rate at which it does work. |
| **Momentum and Impulse**  Be able to calculate the impulse of a force as a vector and in component form.  Understand and use the concept of linear momentum and appreciate that it is a vector quantity.  Understand and use the impulse-momentum equation.  Understand and use the principle that a system subject to no external force has constant total linear momentum and that this result may be applied in any direction.  Understand the term direct impact and the assumptions made when modelling direct impact collisions.  Be able to apply the principle of conservation of linear momentum to direct impacts within a system of bodies.  Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to a direct impact.  Understand the significance of e=0  Be able to apply Newton's Experimental Law in modelling direct impacts.  Be able to model situations involving direct impact using both conservation of linear momentum and Newton's Experimental Law.  Understand the significance of *e=1*  Understand that when e<1 kinetic energy is not conserved during impacts and be able to find the loss of kinetic energy. |
| **Centre of Mass**  Be able to find the centre of mass of a system of particles of given position and mass.  Know how to locate centre of mass by appeal to symmetry.  Know the positions of the centres of mass of a uniform rod, a rectangular lamina and a triangular lamina.  Be able to find the centre of a mass of a composite body by considering each constituent part as a particle at its centre of mass.  Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body. |

Y13FM Statistics

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| **Sampling**  Be able to explain the importance of sample size in experimental design.  Be able to explain why sampling may be necessary in order to obtain information about a population, and give desirable features of a sample.  Be able to explain the advantage of using a random sample when inferring properties of a population. |
| **Discrete Random Variables including Poisson and Geometric Distribution**  Be able to use probability functions, given algebraically or in tables.  Be able to calculate the numerical probabilities for a distribution.  Be able to draw and interpret graphs representing probability distributions.  Be able to calculate the expectation (mean), E(X), and understand its meaning.  Be able to calculate the variance, Var(X), and understand its meaning.  Be able to use the result E(a+bX)=a+bE(X) and understand its meaning.  Be able to use the result Var(a+bX)= b2Var(X) and understand its meaning.  Be able to find the mean of any linear combination of random variables and the variance of any linear combination of independent random variables.  Recognise situations under which the discrete uniform distribution is likely to be an appropriate model.  Be able to calculate probabilities using a discrete uniform distribution.  Be able to calculate the mean and variance of any given discrete uniform distribution.  Recognise situations under which the binomial distribution is likely to be an appropriate model, and be able to calculate probabilities to use the model.  Know and be able to use the mean and variance of a binomial distribution. Prove these results in particular cases.  Recognise situations under which the Poisson distribution is likely to be an appropriate model.  Recognise situations in which both the Poisson distribution and the binomial distribution might be appropriate models.  Be able to calculate probabilities using a Poisson distribution.  Know and be able to use the mean and variance of a Poisson distribution.  Know that the sum of two or more independent Poisson distributions is also a Poisson distribution.  Recognise situations under which the geometric distribution is likely to be an appropriate model.  Be able to calculate the probabilities within a geometric distribution, including cumulative probabilities.  Be able to use the mean and variance of a geometric distribution. |
| **Bivariate Date**  Understand what bivariate data are and know the conventions for choice of axis for variables in a scatter diagram.  Be able to use and interpret a scatter diagram.  Interpret a scatter diagram produced by software.  Be able to calculate the PMCC from raw data or summary statistics.  Know when it is appropriate to carry out a hypothesis test using Pearson’s product moment correlation coefficient.  Be able to carry out hypothesis tests using the PMCC and tables of critical values or the *p*-value from software.  Use the PMCC as an effect size.  Be able to calculate Spearman's rank correlation coefficient from raw data or summary statistics.  Be able to carry out hypothesis tests using Spearman's rank correlation coefficient and tables of critical values or the output from software.  Decide whether a test based on *r* or *rs* may be more appropriate, or whether neither is appropriate.  Be able to calculate the equation of the least squares regression line using raw data or summary statistics.  Be able to use the regression line as a model to estimate values and know when it is appropriate to do so.  Know the meaning of the term residual and be able to calculate and interpret residuals.  Be able to calculate the equation of the two least squares regression lines, y on x and x on y*, using raw data or summary statistics.*  Be able to use either regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so.  Check how well the model fits the data.  Know the relationship between the two regression lines and when to use one rather than the other.  Be able to use the correct regression line to estimate the expected value of one variable for a given value of the other and know when it is appropriate to do so. |
| **Chi-Squared Tests**  Be able to interpret bivariate categorical data in a contingency table.  Be able to apply the test (chi-squared) to a contingency table.  Be able to interpret the results of a test using tables of critical values or the output from software.  Be able to carry out a test for goodness of fit of a uniform, binomial or Poisson model.  Be able to interpret the results of a test using tables of critical values or the output from software. |