Y12FM Pure maths

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| **Complex Numbers**  Understand the language of complex numbers.  Be able to solve any quadratic equation with real coefficients.  Know that the complex roots of polynomial equations with real coefficients occur in conjugate pairs. Be able to solve cubic or quartic equations with real coefficients.  Be able to add, subtract, multiply and divide complex numbers given in the form *x+yi*, *x* and *y* real.  Understand that a complex number is zero if and only if both the real and imaginary parts are zero.  Be able to represent a complex number in modulus-argument form. Be able to convert between the forms *z=x+yi* and *z*=*r*( cos *θ* + i sin *θ*) where *r* is the modulus and *θ* is the argument of the complex number.  Be able to multiply and divide complex numbers in modulus-argument form.  Be able to represent and interpret complex numbers and their conjugates on an Argand diagram.  Be able to represent the sum, difference, product and quotient of two complex numbers on an Argand diagram.  Be able to represent and interpret sets of complex numbers as loci on an Argand diagram. |
| **Matrices**  Be able to add, subtract and multiply conformable matrices, and to multiply a matrix by a scalar.  Understand and use the zero and identity matrices, understand what is meant by equal matrices.  Know that matrix multiplication is associative but not commutative.  Be able to find the matrix associated with a linear transformation and vice-versa.  Understand successive transformations in 2-D and the connection with matrix multiplication.  Understand the language of vectors in two dimensions and three dimensions.  Know the meaning of, and be able to find, invariant points and invariant lines for a linear transformation.  Be able to calculate the determinant of a 2×2 matrix and a 3×3 matrix.  Know the meaning of the terms singular and non-singular as applied to matrices.  Know that the magnitude of the determinant of a 2×2 matrix gives the area scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.  Know that the magnitude of the determinant of a 3×3 matrix gives the volume scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.  Know that det(**MN**)=det **M** ×det **N** and the corresponding result for scale factors of transformations.  Understand what is meant by an inverse matrix.  Be able to calculate the inverse of a non-singular 2×2 matrix or 3×3 matrix.  Be able to use the inverse of a non-singular 2×2 or 3×3 matrix. Relate the inverse matrix to the corresponding inverse transformation.  Understand and use the product rule for inverse matrices. |
| **Vectors**  Know how to calculate the scalar product of two vectors, and be able to use the two forms of the scalar product to find the angle between two vectors.  Be able to form and use the vector and cartesian equations of a plane. Convert between vector and cartesian forms for the equation of a plane.  Know that a vector which is perpendicular to a plane is perpendicular to any vector in the plane.  Know the different ways in which three distinct planes can be arranged in 3-D space.  Be able to solve three linear simultaneous equations in three variables by use of the inverse of the corresponding matrix.  Interpret the solution or failure of solution geometrically in terms of the arrangement of three planes.  Be able to find the intersection of three planes when they meet in a point.  Know that the angle between two planes can be found by considering the angle between their normals. |
| **Proof**  Be able to prove mathematical results by deduction and exhaustion, and disprove false conjectures by counter example.  Induction: Be able to construct and present a proof using mathematical induction for given results for a formula for the *n*th term of a sequence, the sum of a series or the *n*th power of a matrix. |
| **Algebra and Series**  Understand and use the relationships between the roots and coefficients of quadratic, cubic and quartic equations.  Be able to form a new equation whose roots are related to the roots of a given equation by a linear transformation.  Be able to use standard formulae for , and and the method of differences to sum series. |

Y12FM Modelling with Algorithms

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| **Algorithms**  Understand that an algorithm is a finite sequence of operations for carrying out a procedure or solving a problem.  Understand that an algorithm can be the basis for a computer program.  Be able to interpret and apply algorithms presented in a variety of formats.  Be able to repair, develop and adapt given algorithms.  Understand and be able to use the basic ideas of algorithmic complexity and be able to analyse the complexity of given algorithms.  Know that complexity can be used, among other things, to compare algorithms.  Understand that algorithms can sometimes be proved correct or incorrect.  Understand and know the importance of heuristics.  Know and be able to use the quick sort algorithm.  Be able to apply other sorting algorithms which are specified.  Be able to count the number of comparisons and/or swaps needed in particular applications of sorting algorithms, and relate this to complexity.  Be able to reason about a given sorting algorithm.  Know and be able to use first fit and first fit decreasing packing algorithms and full bin strategies.  Be able to count the number of comparisons needed in particular applications of packing algorithms, and relate this to complexity. |
| **Networks**  Understand and be able to use graphs and associated language.  Be able to model problems by using graphs.  Understand that a network is a graph with weighted arcs.  Be able to model problems by using networks.  Be able to solve minimum connector problems using Kruskal’s and Prim’s algorithms.  Model shortest path problems and solve using Dijkstra’s algorithm.  Know and use the fact that Kruskal’s, Prim’s and Dijkstra’s algorithms have quadratic complexity.  Model precedence problems with an activity-on-arc network.  Use critical path analysis and be able to interpret outcomes, including implications for criticality.  Be able to analyse float (total, independent and interfering), resourcing and scheduling.  Be able to use a network to model a transmission system.  Be able to specify a cut and calculate its capacity.  Understand and use the maximum flow/minimum cut theorem.  Understand that network algorithms can be explored, understood and tested in cases in which the algorithm can be run by hand, but for practical problems the algorithm needs to be formulated in a way suitable for computing power to be applied. |
| **Linear Programming**  Understand and use the language associated with linear programming.  Be able to identify and define variables from a given problem.  Be able to formulate a problem as a linear program.  Be able to recognise when an LP is in standard form.  Be able to use slack variables to convert an LP in standard form to augmented form.  Recognise when an LP requires an integer solution.  Be able to formulate a range of network problems as LPs.  Be able to graph inequalities in 2-D and identify feasible regions.  Be able to recognise infeasibility.  Be able to solve a 2-D LP graphically.  Be able to consider the effect of modifying constraints or the objective function.  Be able to solve 2-D integer LP problems graphically.  Be able to use a visualisation of a 3-D LP to solve it.  Be able to reduce a 3-D LP to a 2-D LP when one constraint is an equality.  Be able to use the simplex algorithm on an LP in augmented form.  Understand the geometric basis for the simplex method.  Recognise that if an LP includes constraints then the two-stage simplex method may be used; understand how this method works and be able to set up the initial tableau in such cases.  Be able to reformulate an equality constraint as a pair of inequality constraints.  Recognise that if an LP has variables which may take negative values or requires the objective function to be minimised then some initial reformulation is required before the simplex algorithm may be applied.  Understand that some LPs can be solved using graphical techniques or the simplex method, but for practical problems computing power needs to be applied.  Know that a spreadsheet LP solver routine, or other software, can solve an LP given in standard form or, in some cases, in non-standard form.  Be able to interpret the output from a spreadsheet optimisation routine, or other software, for the simplex method or ILPs.  Shortest path. |